INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL

MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 2

Tuesday 8 May 2001 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all five questions from Section A and one question from Section B.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures, as appropriate.

- Write the make and model of your calculator on the front cover of your answer booklets *e.g.* Casio *fx-9750G*, Sharp EL-9400, Texas Instruments TI-85.

You are advised to start each new question on a new page. A correct answer with **no** *indication of the method used will usually receive* **no** *marks. You are therefore advised to show your working. (If graphs from a graphic display calculator are being used to find solutions, you should sketch these graphs as part of your answer.)*

SECTION A

Answer all ***five*** *questions from this section.*

1. *[Maximum mark: 17]*

The sets A , B and C are subsets of U . They are defined as follows:

U = {positive integers less than 16}

A = {prime numbers}

B = {factors of 36}

C = {multiples of 4}

(a) List the elements (if any) of the following:

(i) A ;

(ii) B ;

(iii) C ;

(iv) $A \cap B \cap C$. *[4 marks]*

(b) (i) Draw a Venn diagram showing the relationship between the sets U , A , B and C .

(ii) Write the elements of sets U , A , B and C in the appropriate places on the Venn diagram. *[4 marks]*

(c) From the Venn diagram, list the elements of each of the following

(i) $A \cap (B \cup C)$;

(ii) $(A \cap B)'$;

(iii) $(A \cap B)' \cap C$. *[3 marks]*

(This question continues on the following page)

(Question 1 continued)

(d) Find the probability that a number chosen at random from the universal set U will be

(i) a prime number ;

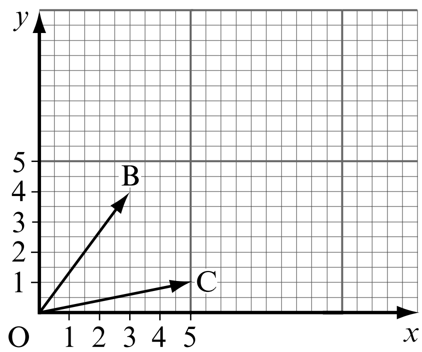
(ii) a prime number, but **not** a factor of 36 ;

(iii) a factor of 36 or a multiple of 4, but **not** a prime number ;

(iv) a prime number, given that it is a factor of 36 . *[6 marks]*

2. *[Maximum mark: 7]*

The following diagram shows the points O, B and C, and the vectors $\overrightarrow{OB}$ and $\overrightarrow{OC}$.



(a) Write each of the following vectors in the form $\begin{pmatrix} x \\ y \end{pmatrix}$.

(i) $\overrightarrow{OB}$;

(ii) $\overrightarrow{OC}$. *[2 marks]*

(b) (i) Copy the above diagram onto graph paper. Given that $\overrightarrow{OB} = \overrightarrow{CP}$, plot the point P and write its coordinates on your graph.

(ii) Join the points O, B, P and C to form a quadrilateral. What type of quadrilateral is OBPC? *[3 marks]*

(c) Calculate the length of $\overrightarrow{OP}$. *[2 marks]*

3. *[Maximum mark: 10]*

ABCD is a trapezium with AB = CD and [BC] parallel to [AD] . AD = 22 cm , BC = 12 cm , AB = 13 cm .

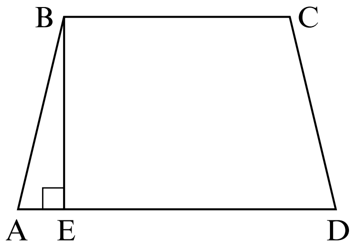


diagram not to scale

(a) Show that AE = 5 cm . *[2 marks]*

(b) Calculate the height BE of the trapezium. *[2 marks]*

(c) Calculate

(i) $B\widehat{A}E$;

(ii) $B\widehat{C}D$. *[3 marks]*

(d) Calculate the length of the diagonal [CA] . *[3 marks]*

4. *[Maximum mark: 18]*

(i) The following is a currency conversion table:

	FFR	USD	JPY	GBP
French Francs (FFR)	1	p	q	0.101
US Dollars (USD)	6.289	1	111.111	0.631
Japanese Yen (JPY)	0.057	0.009	1	0.006
British Pounds (GBP)	9.901	1.585	166.667	1

For example, from the table 1 USD = 0.631 GBP . Use the table to answer the following questions.

(a) Find the values of p and q . *[2 marks]*

(b) Mireille wants to change money at a bank in London.

(i) How many French Francs (FFR) will she have to change to receive 140 British Pounds (GBP)?

(ii) The bank charges a 2.4% commission on all transactions. If she makes this transaction, how many British Pounds will Mireille actually receive from the bank? *[4 marks]*

(c) Jean invested 5000 FFR in Paris at 8% simple interest per annum. Paul invested 800 GBP in London at 6% simple interest per annum.

(i) How much interest in FFR did Jean earn after 4 years?

(ii) How much interest in US Dollars did Paul earn after 4 years?

(iii) Who had earned more interest after 4 years?

(iv) Explain your reasoning in part (c) (iii). *[7 marks]*

(ii) Takaya invested 1000 JPY at 6.3% simple interest for 15 years. Morimi invested 900 JPY at 6.3% interest compounded annually for 15 years. Who had more money at the end of the 15th year? Justify your answer **clearly**. *[5 marks]*

5. *[Maximum mark: 18]*

(i) The number (n) of bacteria in a colony after h hours is given by the formula $n = 1200\ (3^{0.25h})$. Initially, there are 1200 bacteria in the colony.

(a) Copy and complete the table below, which gives values of n and h . **Give your answers to the nearest hundred.**

time in hours (h)	0	1	2	3	4
no. of bacteria (n)	1200		2100	2700	

[2 marks]

(b) On graph paper, draw the graph of the above function. Use a scale of 3 cm to represent 1 hour on the horizontal axis and 4 cm to represent 1000 bacteria on the vertical axis. Label the graph clearly. *[5 marks]*

(c) Use your graph to answer each of the following showing your method **clearly**.

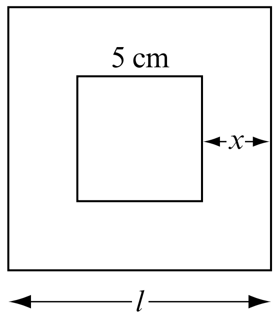
(i) How many bacteria would there be after 2 hours and 40 minutes? Give your answer to the nearest hundred bacteria.

(ii) After how long will there be approximately 3000 bacteria? Give your answer to the nearest 10 minutes. *[4 marks]*

(This question continues on the following page)

(Question 5 continued)

(ii) A picture is in the shape of a square of side 5 cm . It is surrounded by a wooden frame of width x cm , as shown in the diagram below.



The length of the wooden frame is l cm , and the area of the wooden frame is A cm ^{2} .

(a) Write an expression for the length l in terms of x . *[1 mark]*

(b) Write an expression for the area A in terms of x . *[2 marks]*

(c) If the area of the frame is 24 cm^2 , find the value of x . *[4 marks]*

SECTION B

*Answer **one** question from this section.*

Matrices and Graph Theory

6. *[Maximum mark: 30]*

(i) A T-shirt vendor purchased the following numbers of T-shirts for the last World Cup football finals.

Brasil: 125 small, 125 medium, 150 large and 200 extra large

France: 200 small, 200 medium, 225 large and 250 extra large

Small T-shirts cost the vendor USD 7 each, medium and large T-shirts cost him USD 9 each and extra large T-shirts cost USD 10 each.

(a) (i) Construct a 2×4 matrix $\boldsymbol{T}$ to represent the number of T-shirts purchased for Brasil and France in each of the four sizes. Label the matrix clearly.

(ii) Explain the significance of the matrix $\boldsymbol{C}$ below:

$$\boldsymbol{C} = \begin{pmatrix} 7 \\ 9 \\ 9 \\ 10 \end{pmatrix}.$$

[3 marks]

(b) Given that $\boldsymbol{TC} = \begin{pmatrix} 5350 \\ \boldsymbol{a} \end{pmatrix}$,

(i) showing your method, calculate the value of $\boldsymbol{a}$.

(ii) what is the significance of this value? *[4 marks]*

(c) The matrix below represents the number of each size of T-shirt left after the game.

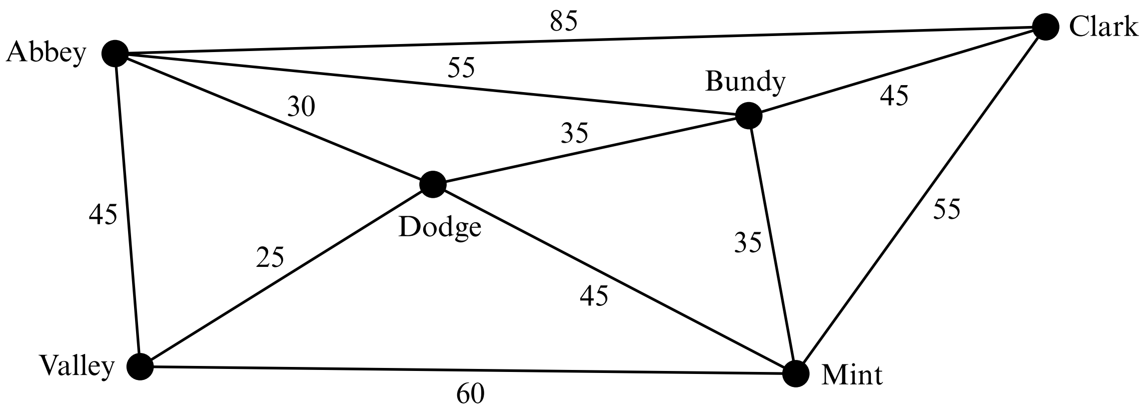
$$\begin{array}{c c} & \begin{array}{cccc} \text{small} & \text{medium} & \text{large} & \text{extra large} \end{array} \\ \begin{array}{c} \text{Brasil} \\ \\ \text{France} \end{array} & \begin{pmatrix} 50 & 50 & 45 & 50 \\ \\ 25 & 60 & 25 & 40 \end{pmatrix} \end{array}$$

Find the value of all of the T-shirts that the vendor had left after the game. *[2 marks]*

(This question continues on the following page)

(Question 6 continued)

(ii) The graph below shows the roads joining a number of small country towns. The distances between the towns shown on the graph are in kilometres.



(a) From the graph, construct a tree that connects all six towns. Clearly label the vertices. *[2 marks]*

(b) Describe a path that goes from Valley to Clark that is exactly 155 kilometres long. *[2 marks]*

(c) In the context of the problem, what is represented by

(i) the vertices of the graph?

(ii) the edges of the graph? *[2 marks]*

(d) What is the degree of the vertex Dodge? *[1 mark]*

(e) A road inspection team has to inspect all the roads shown on the graph.

(i) It is not possible for them to start and finish in the same town and traverse each road only once. Explain why.

(ii) Describe a path that starts and finishes in different towns and that traverses each road only once. *[4 marks]*

(f) Construct an adjacency matrix for this graph. *[3 marks]*

(This question continues on the following page)

(Question 6 continued)

(iii) Brad and Janet are playing a zero sum game. Brad has a choice of ringing one of three bells labelled A , B and C . Janet has a choice of ringing one of three bells labelled D , E and F . The following matrix shows how much Brad may win each time (*payoff matrix* for Brad).

$$\begin{array}{cc} & \text{Janet} \\ & \begin{array}{ccc} \text{D} & \text{E} & \text{F} \end{array} \\ \text{Brad} \begin{array}{c} \text{A} \\ \text{B} \\ \text{C} \end{array} & \begin{pmatrix} -4 & 6 & 5 \\ 5 & -7 & 3 \\ -8 & 0 & 6 \end{pmatrix} \end{array}$$

(a) Explain the consequences for both players if Brad rings bell B and Janet rings bell E . *[1 mark]*

Both players want to use a strategy that will minimise their losses (*play safe strategy*).

(b) What is the *play safe strategy* for both players? Show your method clearly. *[3 marks]*

(c) (i) What is Brad's best strategy if he assumes that Janet will play safe?

(ii) What is Janet's best strategy if she assumes that Brad will play safe?

(iii) What is the consequence if both players select their strategy based on the assumption that the other player will play safe? *[3 marks]*

Further Statistics and Probability

7. *[Maximum mark: 30]*

(i) A machine dispenses *Mighty Flakes* cereal into boxes so that the weight w of each box of *Mighty Flakes* is normally distributed with a mean of 750 grams and a standard deviation of 10 grams. The machine **does not** reject any box whose weight is within two standard deviations of the mean.

(a) Calculate the probability that a box of *Mighty Flakes* cereal dispensed in such a way will weigh

(i) over 765 grams ;

(ii) under 725 grams ;

(iii) between 725 and 765 grams . *[6 marks]*

(b) (i) What is the probability that the machine will reject a box of *Mighty Flakes*?

(ii) Out of 1000 boxes, how many should be rejected by the machine? *[4 marks]*

(c) One such machine was being tested to see if it was operating properly, that is, that its weights of boxes were normally distributed with the above mean and standard deviation.

One hundred boxes were tested and the following data was collected:

61 boxes weighed from 740 to 760 grams
30 boxes weighed from 730 to 739 grams or from 761 to 770 grams
9 boxes weighed either below 730 grams or above 770 grams

Using a 5% level of significance, perform a Chi Squared Test on the above data to see if it does fit the above normal distribution. You may use the following approximations for a normal distribution curve:

Values of Z	Percentage of normal curve between these values
$-1 \le Z \le 1$	68
$-2 \le Z < -1$ or $1 < Z \le 2$	27
$Z < -2$ or $Z > 2$	05

(This question continues on the following page)

(Question 7 continued)

(i) Write down the null and alternate hypotheses. *[2 marks]*

(ii) Copy and complete the table for observed and expected frequencies:

Values of Z	f_o	f_e	$f_e - f_o$	$(f_e - f_o)^2$
$-1 \le Z \le 1$				
$-2 \le Z < -1$ or $1 < Z \le 2$				
$Z < -2$ or $Z > 2$				

[6 marks]

(iii) Calculate the value of the chi-squared statistic. *[3 marks]*

(iv) Write down the critical value of chi-squared, and hence decide if the data does fit the above normal distribution. *[3 marks]*

(ii) Ten students were asked for their average grade at the end of their last year of high school and their average grade at the end of their last year at university. The results were put into a table as follows:

Student	High School grade, x	University grade, y
1	90	3.2
2	75	2.6
3	80	3.0
4	70	1.6
5	95	3.8
6	85	3.1
7	90	3.8
8	70	2.8
9	95	3.0
10	85	3.5
Total	835	30.4

(a) Given that $s_x = 8.96$, $s_y = 0.610$ and $s_{xy} = 4.16$, find the correlation coefficient r, giving your answer to two decimal places. *[2 marks]*

(b) Describe the correlation between the high school grades and the university grades. *[2 marks]*

(c) Find the equation of the regression line for y on x in the form $y = ax + b$. *[2 marks]*

Introductory Differential Calculus

8. *[Maximum mark: 30]*

The velocity, v ms^{-1}, of a kite, after t seconds, is given by

$$v = t^3 - 4t^2 + 4t, \qquad 0 \le t \le 4.$$

(a) What is the velocity of the kite after

(i) one second?

(ii) half a second? *[2 marks]*

(b) Calculate the values of a and b in the table below.

t	0	0.5	1	1.5	2	2.5	3	3.5	4
v	0			a	0	0.625	b	7.88	16

[2 marks]

(c) (i) Find $\dfrac{\mathrm{d}v}{\mathrm{d}t}$ in terms of t. Find the value of t at the local maximum and minimum values of the function.

(ii) Explain what is happening to the function at its local maximum point. Write down the gradient of the tangent to its curve at this point. *[8 marks]*

(d) On graph paper, draw the graph of the function $v = t^3 - 4t^2 + 4t$, $0 \le t \le 4$. Use a scale of 2 cm to represent 1 second on the horizontal axis and 2 cm to represent 2 ms^{-1} on the vertical axis. *[5 marks]*

(e) Describe the motion of the kite at different times during the first 4 seconds. Write down the intervals corresponding to changes in motion. *[3 marks]*

The acceleration, a, of a second kite is given by the equation $a = 4t - 3$, $0 \le t \le 4$.

(f) After 1 second, the velocity, u, of this kite, is 3 ms^{-1}. Write the equation for u in terms of t. *[4 marks]*

(g) After how many seconds does the motion of the second kite change from deceleration to acceleration? *[2 marks]*

(h) Is this kite accelerating or decelerating at t = 0.5 seconds? Justify your answer. *[2 marks]*

(i) During which time interval do **both** kites increase their velocity? *[2 marks]*